# Multi-Scale Correlation Analysis between Tests and Simulations for Automobile Crashes

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**General Dynamics**

**Advanced Information Systems**

GENERAL DYNAMICS

# Introduction

- Rapid development of computational modeling and simulation
- Model types
  - Lumped-parameter models
  - Rigid multi-body models
  - Finite element (FE) models
- Model attributes
  - Different levels of abstraction
  - Characteristics of the system at different scales
  - Various applications
- Model validation lagging behind the model development
  - Test data limited or even unavailable
  - Ineffective execution of the model validation

# Model Validation

- Model Validation
  - To determine the degree to which a computer simulation is an accurate representation of the real world
  - Necessary for a model to be useful for practical analysis and design
- Models exhibiting different characteristics at different scales
- Model validation to be performed at multiple scales
- Validation methods
  - Test-simulation correlation
  - Visual inspection
  - Quantitative and objective evaluations

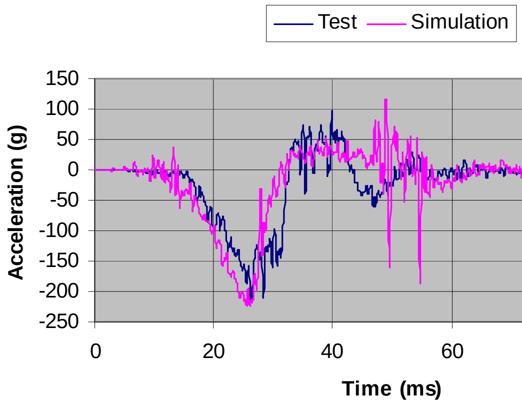
# Teat-simulation Correlation

- Comparison between simulation results and test data for respective responses
- Elements involved in comparison
  - Items to be compared
  - Manner in which to make the comparison
  - Metrics by which to measure the comparison
- Comparison in terms of
  - Point-to-point
  - Peak values
  - Injury criteria
  - Corridor
- ✓ Point-point comparison at multiple scales

# Impact Responses and Wavelets

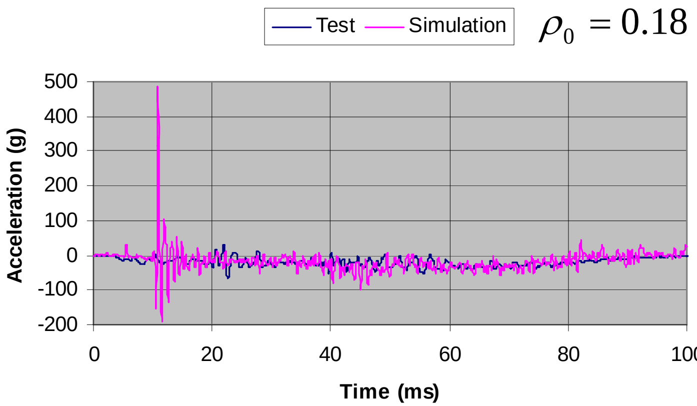
- Impact responses
  - Short time duration
  - Transient, non-stationary
  - Contaminated with noise
- Test-simulation correlation
  - Conventional correlation analysis
  - Poor overall agreement
  - Sound agreement possibly existing in a certain frequency range
- Wavelets
  - Localized in the frequency and time domains, matching with the major features of impact or crash responses
  - To be used in the model validation

$\rho_0 = 0.74$




**Accelerations at engine bottom location**

$\rho_0 = 0.18$

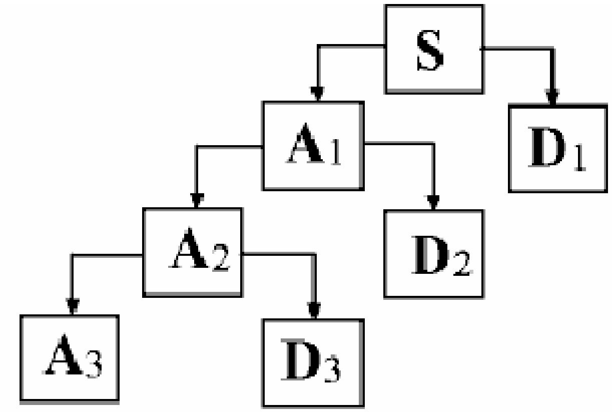


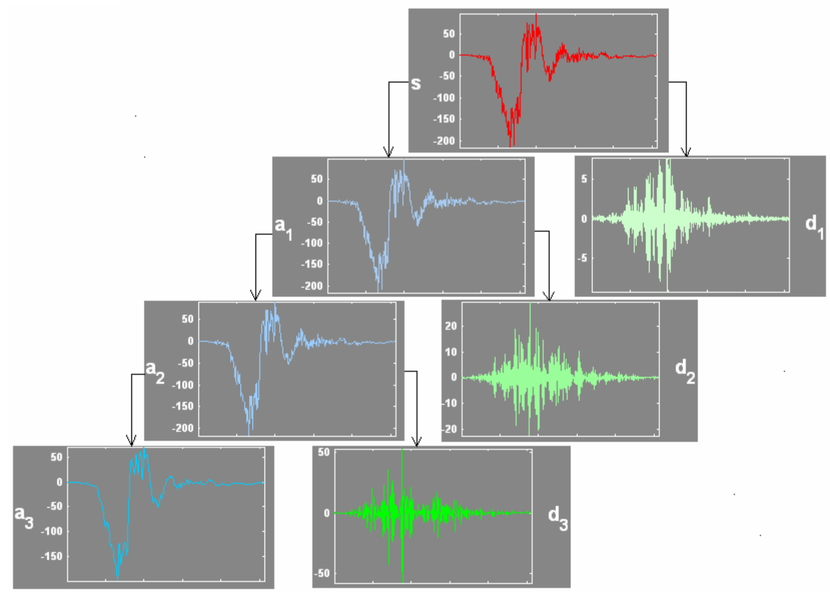

**Accelerations at right-rear cross member location**

# Wavelet Decomposition

**Decomposition of a signal on a wavelet basis:**

$$
\begin{aligned}
S &= A_1 + D_1 \\
&= A_2 + D_2 + D_1 \\
&= \cdots \\
&= A_J + \sum_{j \le J} D_j
\end{aligned}
$$




$A_j$ **: the approximation at level j**

$D_j$ **: the detail at level j**

**Dyadic wavelet decomposition**

**— a multi-resolution analysis**

# Multi-scale Correlation Analysis Based on Wavelet Decompositions

- Conventional correlation analysis inappropriate for impact responses
- Correlation analysis based on wavelet decompositions
  - Two signals decomposed at certain level
  - Approximations retaining low frequency contents, describing underlying gross motions
  - Treating approximations as deterministic, the correlation coefficient between them defined as

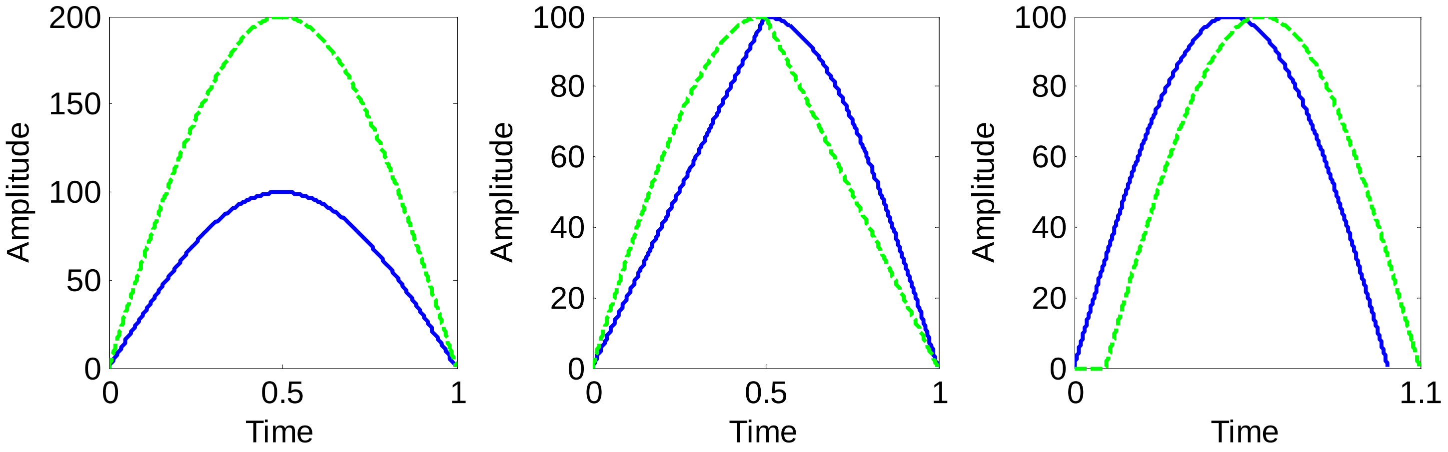
$$\rho_a^l(\tau) = \frac{R_{x_a y_a}^l(\tau) - \mu_{x_a}^l \mu_{y_a}^l}{\sqrt{[R_{x_a x_a}^l(0) - (\mu_{x_a}^l)^2][R_{y_a y_a}^l(0) - (\mu_{y_a}^l)^2]}}$$

  - Accounting for time shift between them

$$\rho_{am}^l = \rho_a^l(\tau_m) = \max_{\tau}\{\rho_a^l(\tau)\}$$

- Correlation analysis at multiple scales

# Inadequacy of Correlation Analysis




Three cases for which correlation analysis unable to tell differences:

- Amplitude difference
- Shape difference
- Time shift

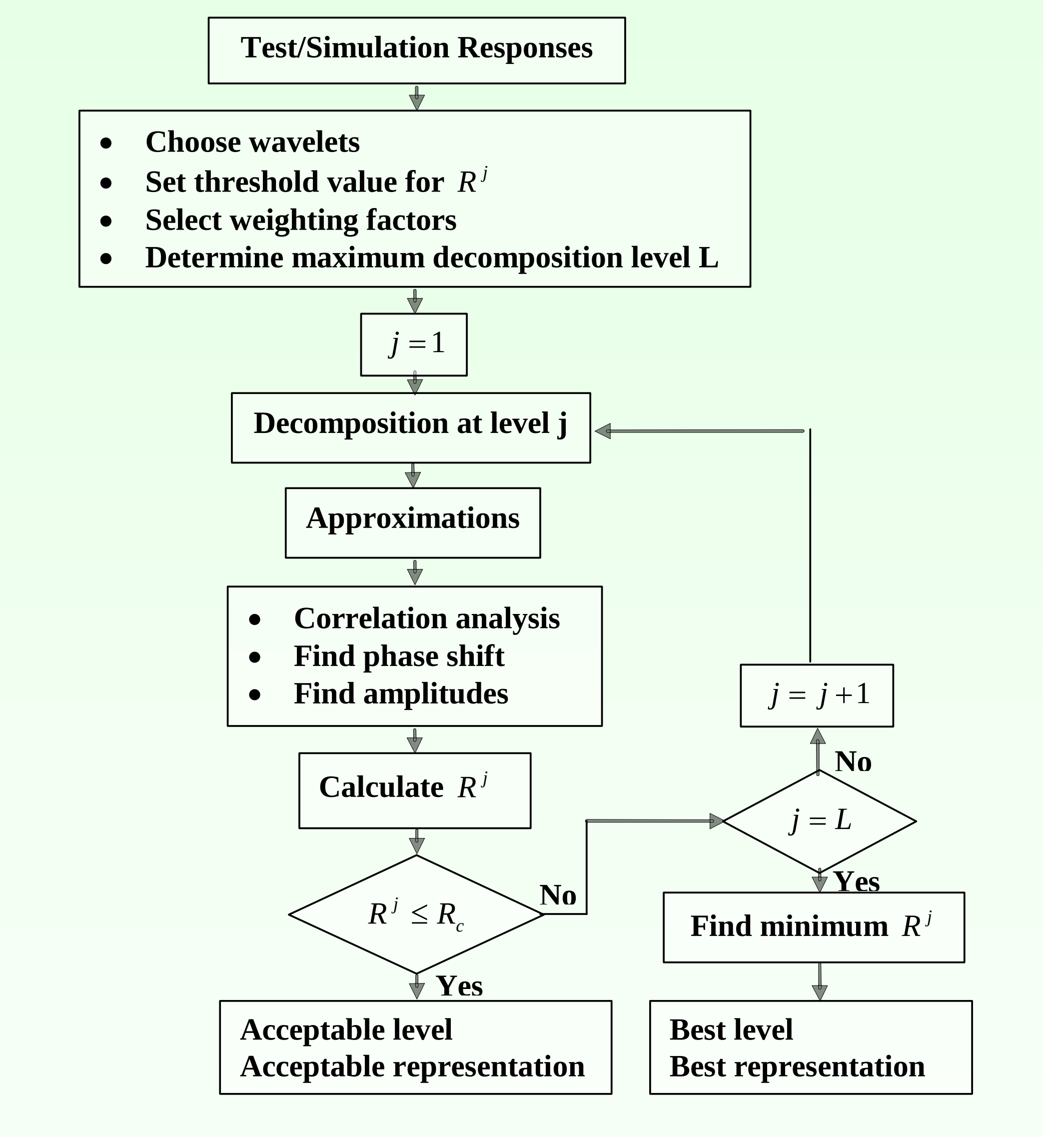
# Multi-scale Validation Metric

- A metric to measure overall agreement in shape, phase, and amplitude

$$R^l = [\alpha_1 (1 - \rho_{am}^l) + \alpha_2 \left| \frac{\tau_m^l}{T} \right| + \alpha_3 \left| \frac{A_t^l - A_s^l}{A_t^l} \right| ] \times 100\%$$

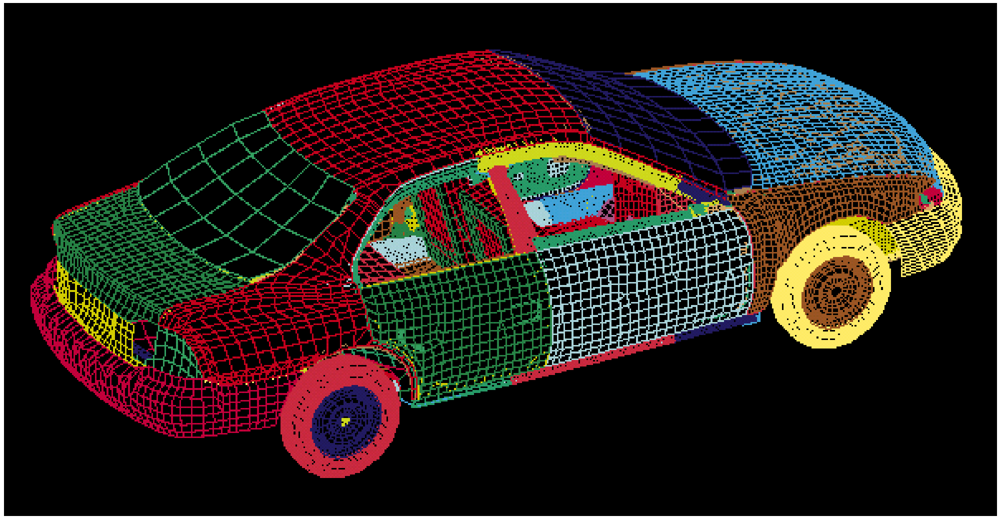
- Dyadic wavelet decomposition as a multi-resolution analysis
- Evaluating the agreement between tests and simulations at different scales and frequency bands—multi-scale validation

# Multi-scale Validation Procedure

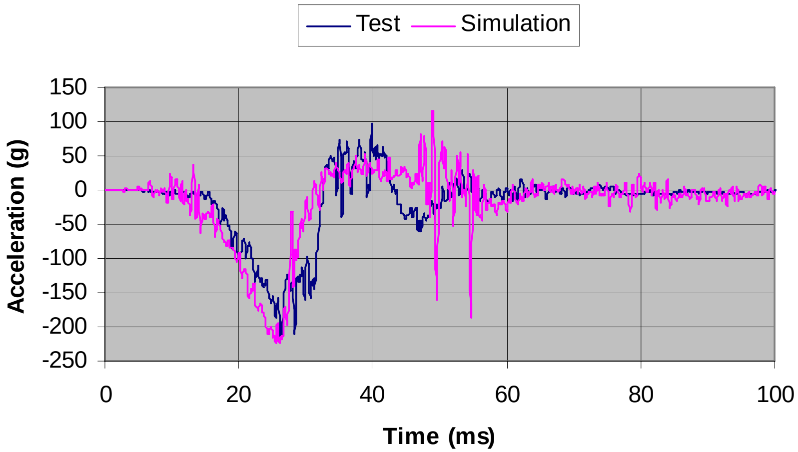


# Test-simulation Correlation
## A Case Study

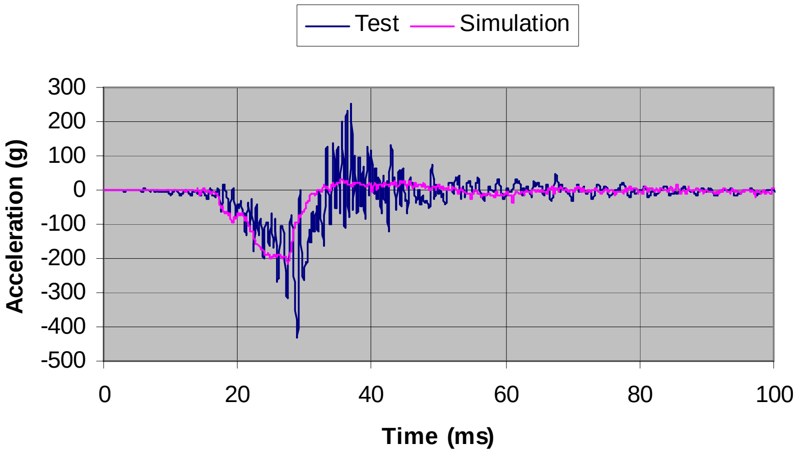
- Test data: a full front impact test of a two-door 1997 Honda Accord (NHTSA database test No. 2475)
- Simulation: an FE 1997 Honda Accord crash model
- Acceleration responses at the locations of engine bottom, engine top, left- and right-rear cross members



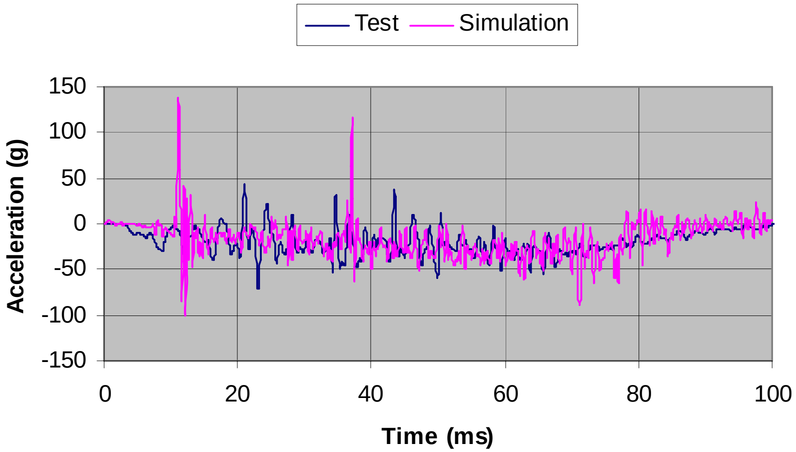
# Acceleration Responses from Test and Simulation



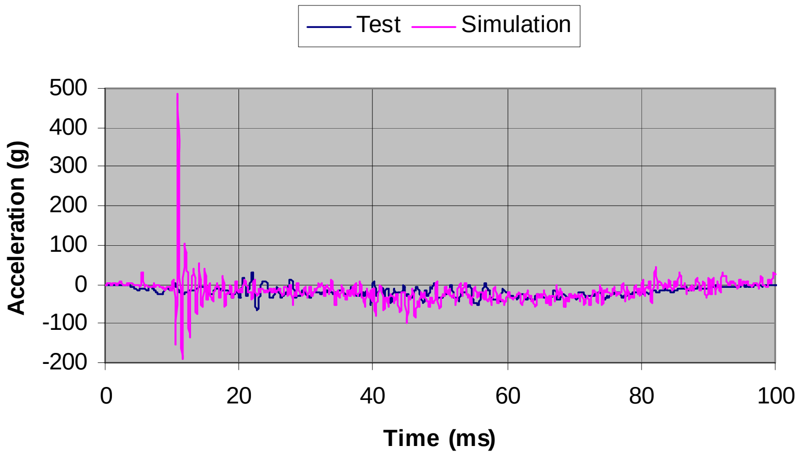

Engine bottom




Engine top




LRCM

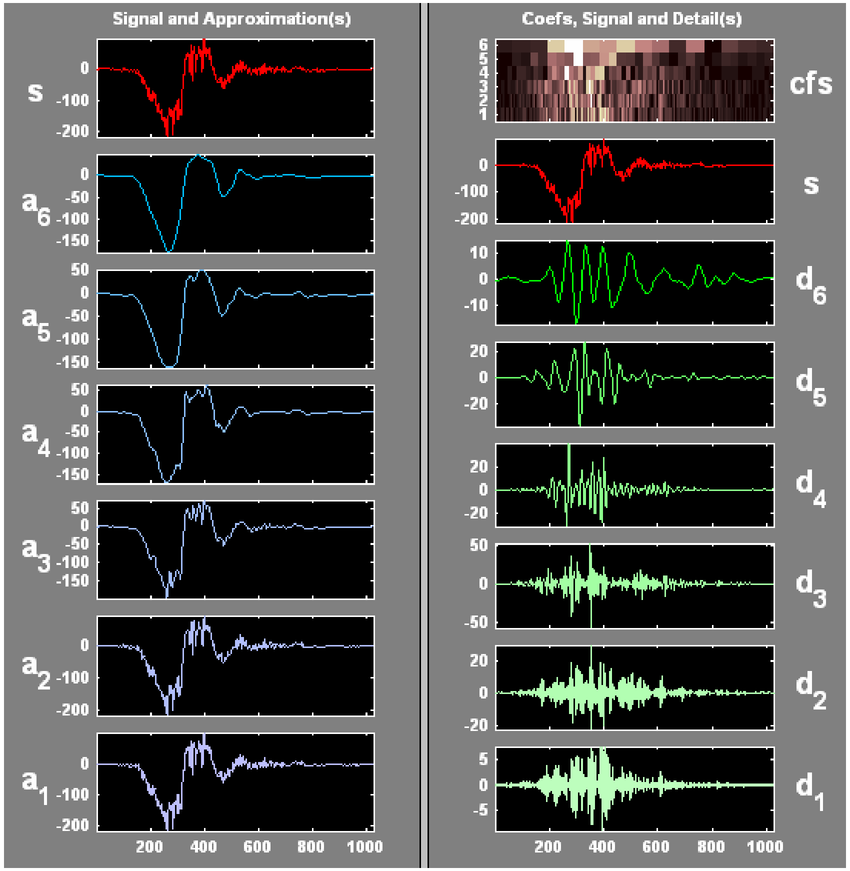



RRCM

# Wavelet Decomposition

On the fourth-order Daubechies wavelet basis, decomposing at level 6

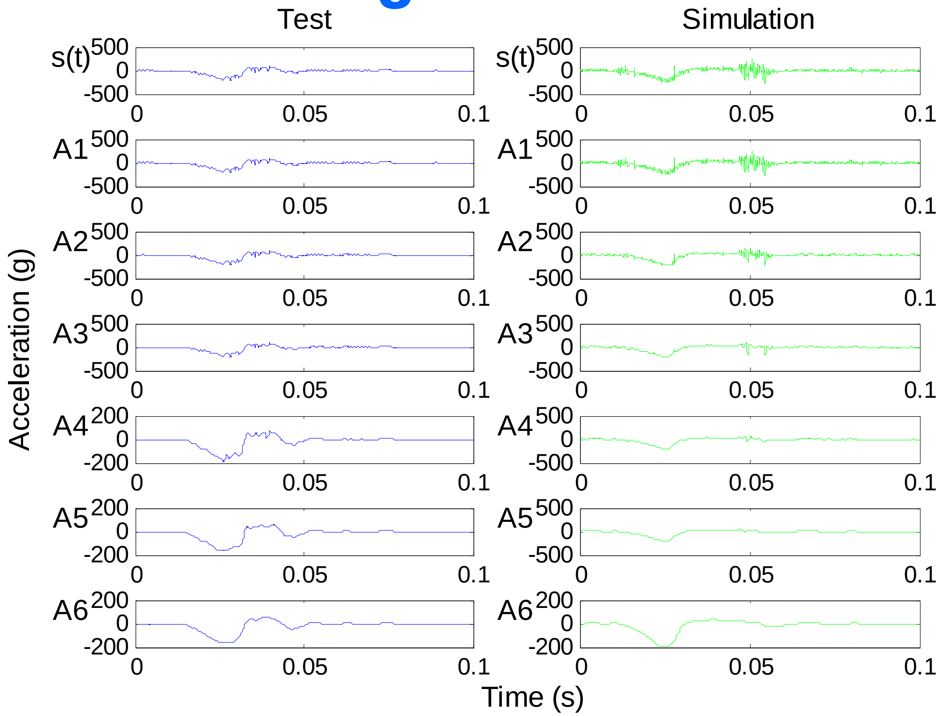
$$S = A_6 + D_6 + D_5 + D_4 + D_3 + D_2 + D_1$$



# Test-simulation Comparison

- The approximation at each level: the gross motion viewed at respective scale
- Correlation between the two approximations at each level
- Peak-time shift between the two approximations
- Amplitude difference between the approximations
- Evaluation with the metric to measure the overall agreement in shape, phase, and amplitude

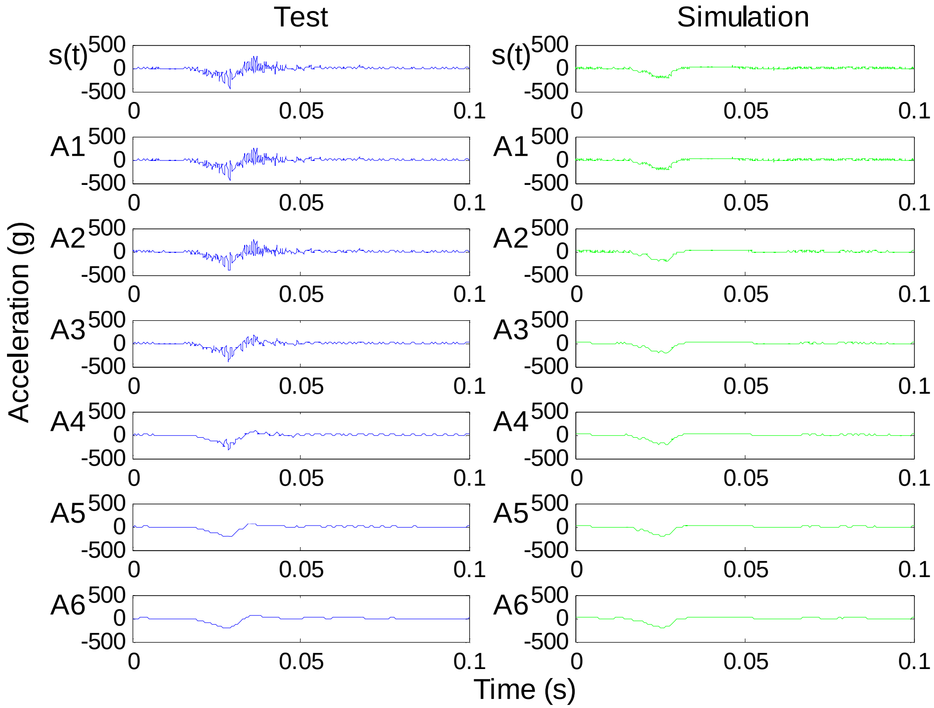
# Original Signal and Approximations
## Engine Bottom




Engine bottom

| | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| $\rho_{am}^j$ | 0.74 | 0.81 | 0.85 | 0.88 | 0.90 | 0.93 | 0.92 |
| $\tau_m^j$ | -2.3 | -2.3 | -2.3 | -1.9 | -1.9 | -2.0 | -1.3 |
| $A_t^j$ | 216 | 216 | 217 | 200 | 170 | 162 | 176 |
| $A_s^j$ | 270 | 226 | 224 | 212 | 215 | 199 | 162 |

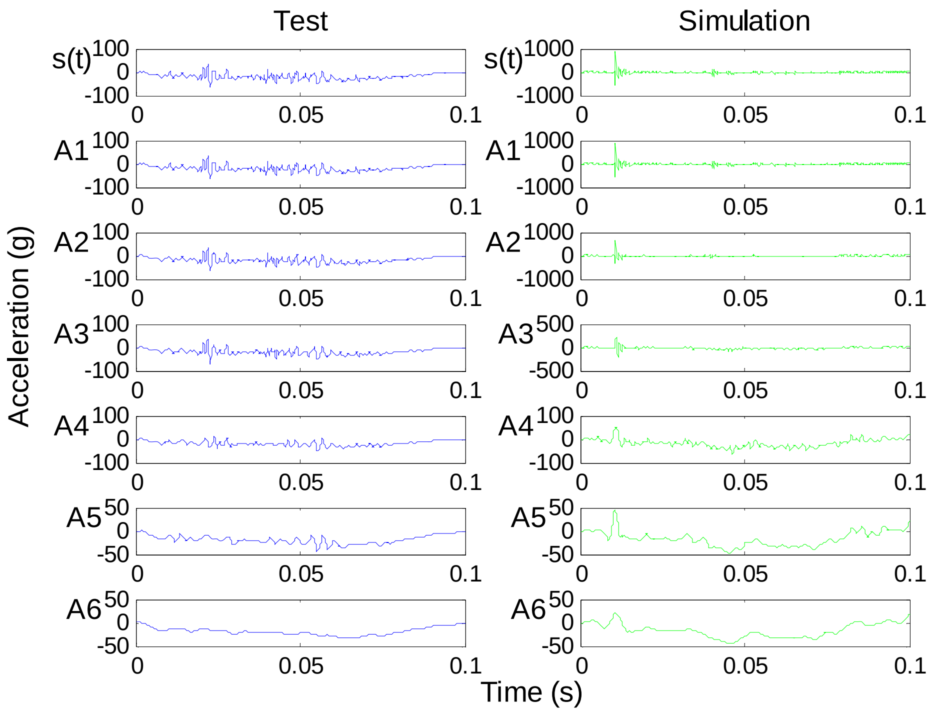
# Original Signal and Approximations
## Engine Top




Engine top

| | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| $\rho_{am}^{j}$ | 0.77 | 0.78 | 0.84 | 0.91 | 0.95 | 0.97 | 0.95 |
| $\tau_m^j$ | -2.1 | -2.1 | -2.2 | -2.2 | -2.2 | -2.3 | -1.2 |
| $A_t^j$ | 432 | 413 | 389 | 334 | 223 | 201 | 215 |
| $A_s^j$ | 221 | 215 | 214 | 207 | 203 | 203 | 162 |

# Original Signal and Approximations
## Left-rear Cross Member

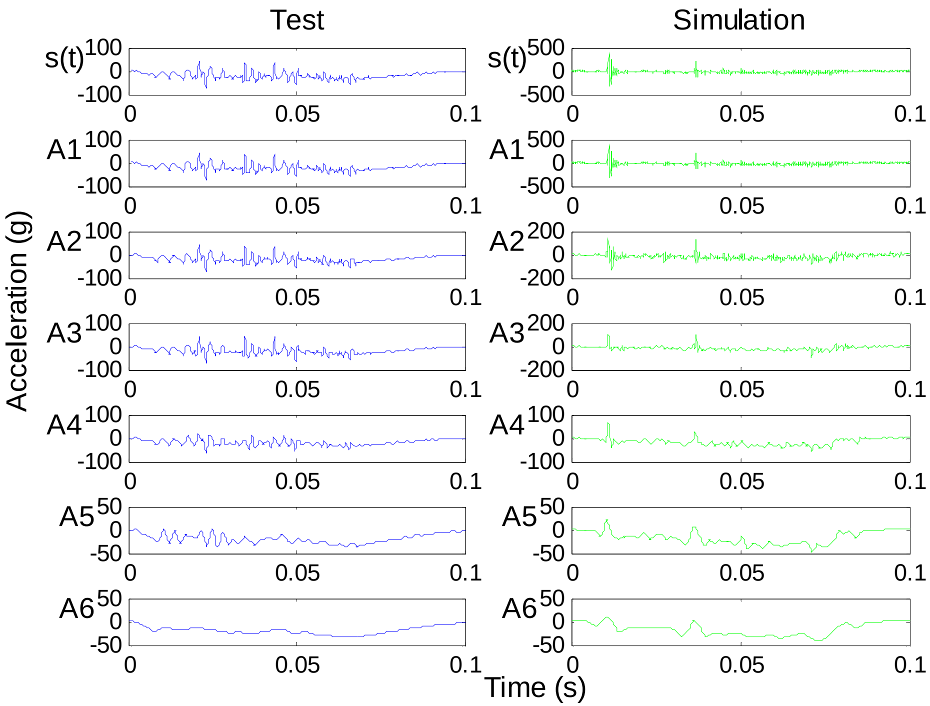



Left-rear cross member

| | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| $\rho_{am}^{j}$ | 0.17 | 0.22 | 0.35 | 0.58 | 0.68 | 0.80 | 0.82 |
| $\tau_{m}^{j}$ | -3.1 | -3.1 | -3.2 | -3.2 | -3.0 | -1.3 | -1.5 |
| $A_t^j$ | 67 | 67 | 70 | 52 | 46 | 32 | 32 |
| $A_s^j$ | 886 | 626 | 205 | 63 | 48 | 45 | 43 |

# Original Signal and Approximations

## Right-rear Cross Member




Right-rear cross member

| | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| $\rho_{am}^{j}$ | 0.18 | 0.29 | 0.37 | 0.48 | 0.66 | 0.77 | 0.87 |
| $\tau_{m}^{j}$ | -2.3 | -2.2 | -2.1 | -2.3 | 0.7 | 0.1 | 1.2 |
| $A_t^j$ | 72 | 73 | 73 | 62 | 38 | 33 | 33 |
| $A_s^j$ | 367 | 137 | 101 | 62 | 47 | 40 | 40 |

# Test-simulation Correlation at Multiple Scales

## Engine Bottom

Engine bottom

| | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| $\rho_{am}^{j}$ | 0.74 | 0.81 | 0.85 | 0.88 | 0.90 | 0.93 | 0.92 |
| $\tau_{m}^{j}$ | -2.3 | -2.3 | -2.3 | -1.9 | -1.9 | -2.0 | -1.3 |
| $A_t^j$ | 216 | 216 | 217 | 200 | 170 | 162 | 176 |
| $A_s^j$ | 270 | 226 | 224 | 212 | 215 | 199 | 162 |

Engine bottom

| $\alpha_1, \alpha_2, \alpha_3$ | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| 0.4, 0.2, 0.4 | 20.7 | 9.94 | 7.87 | 7.69 | 14.9 | 12.3 | 6.55 |
| 0.5, 0.2, 0.3, | 20.8 | 11.4 | 9.06 | 8.28 | 13.2 | 10.8 | 6.50 |
| 0.3, 0.2, 0.5 | 20.6 | 8.49 | 6.67 | 7.09 | 16.5 | 13.9 | 6.60 |
| 0.3, 0.4, 0.3 | 16.1 | 8.03 | 6.48 | 6.24 | 11.6 | 9.74 | 5.24 |

# Test-simulation Comparison at Multiple Scales

## Engine Top

| | Engine top | | | | | | |
|---|---|---|---|---|---|---|---|
| | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
| $\rho_{am}^{j}$ | 0.77 | 0.78 | 0.84 | 0.91 | 0.95 | 0.97 | 0.95 |
| $\tau_{m}^{j}$ | -2.1 | -2.1 | -2.2 | -2.2 | -2.2 | -2.3 | -1.2 |
| $A_t^j$ | 432 | 413 | 389 | 334 | 223 | 201 | 215 |
| $A_s^j$ | 221 | 215 | 214 | 207 | 203 | 203 | 162 |

| $\alpha_1, \alpha_2, \alpha_3$ | Engine top | | | | | | |
|---|---|---|---|---|---|---|---|
| | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
| 0.4, 0.2, 0.4 | 29.2 | 28.4 | 24.7 | 19.2 | 5.97 | 2.28 | 11.9 |
| 0.5, 0.2, 0.3, | 26.6 | 25.8 | 21.7 | 16.3 | 5.54 | 2.51 | 9.99 |
| 0.3, 0.2, 0.5 | 31.8 | 30.9 | 27.6 | 22.1 | 6.41 | 2.06 | 13.9 |
| 0.3, 0.4, 0.3 | 22.4 | 21.8 | 19.0 | 15.0 | 5.03 | 2.29 | 9.29 |

# Test-simulation Correlation at Multiple Scales

## Left-rear Cross Member

Left-rear cross member

| | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| $\rho_{am}^{j}$ | 0.17 | 0.22 | 0.35 | 0.58 | 0.68 | 0.80 | 0.82 |
| $\tau_{m}^{j}$ | -3.1 | -3.1 | -3.2 | -3.2 | -3.0 | -1.3 | -1.5 |
| $A_t^j$ | 67 | 67 | 70 | 52 | 46 | 32 | 32 |
| $A_s^j$ | 886 | 626 | 205 | 63 | 48 | 45 | 43 |

Left-rear cross member

| $\alpha_1, \alpha_2, \alpha_3$ | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| 0.4, 0.2, 0.4 | 522.9 | 363.6 | 103.5 | 26.0 | 14.8 | 24.2 | 21.2 |
| 0.5, 0.2, 0.3, | 408.8 | 288.4 | 90.8 | 28.1 | 17.7 | 22.3 | 19.6 |
| 0.3, 0.2, 0.5 | 636.9 | 438.8 | 116.2 | 24.0 | 12.0 | 26.1 | 22.9 |
| 0.3, 0.4, 0.3 | 392.9 | 273.5 | 78.4 | 20.3 | 11.9 | 18.5 | 16.3 |

# Test-simulation Correlation at Multiple Scales

## Right-rear Cross Member

Right-rear cross member

| | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| $\rho_{am}^{j}$ | 0.18 | 0.29 | 0.37 | 0.48 | 0.66 | 0.77 | 0.87 |
| $\tau_{m}^{j}$ | -2.3 | -2.2 | -2.1 | -2.3 | 0.7 | 0.1 | 1.2 |
| $A_t^j$ | 72 | 73 | 73 | 62 | 38 | 33 | 33 |
| $A_s^j$ | 367 | 137 | 101 | 62 | 47 | 40 | 40 |

Right-rear cross member

| $\alpha_1, \alpha_2, \alpha_3$ | $S(t)$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| 0.4, 0.2, 0.4 | 197.9 | 63.6 | 41.0 | 21.5 | 22.8 | 16.7 | 14.8 |
| 0.5, 0.2, 0.3, | 165.0 | 61.9 | 43.4 | 26.6 | 23.9 | 17.1 | 13.8 |
| 0.3, 0.2, 0.5 | 230.9 | 65.2 | 38.6 | 16.5 | 21.6 | 16.2 | 15.7 |
| 0.3, 0.4, 0.3 | 149.0 | 48.2 | 31.3 | 16.7 | 17.3 | 12.5 | 11.4 |

# Concluding Remarks

- Wavelet transform provides a time-frequency view of automobile impact responses, displaying their features at multiple scales
- Multi-scale test-simulation correlation can be performed based on the approximations at each level
- More complete test-simulation comparison can be achieved with a metric including correlation coefficient, timing shift, and amplitude difference
- Multi-scale analysis allows for the consideration of system characteristics at different levels of accuracy and detail
- Multi-scale validation can be used to determine an acceptable level and the optimal level for a computer model